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A SIMPLE CLOSED-FORM SOLUTION OF A POSITION-FIXING PROBLEM.(U)

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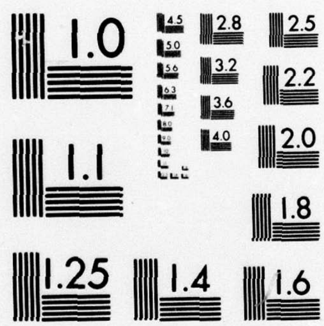

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NRL Report 8115

A Simple Closed-Form Solution of A Position-Fixing Problem

B. H. CANTRELL

Radar Analysis Staff
Radar Division

April 15, 1977

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Report 8115	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A SIMPLE CLOSED-FORM SOLUTION OF A POSITION-FIXING PROBLEM.		5. TYPE OF REPORT & PERIOD COVERED Interim report, on a continuing NRL problem
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ben H. Cantrell	15 Apr 77	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375	10p.	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem R12-18 OMN
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Naval Ship Engineering Center Washington, D.C. 20362		12. REPORT DATE April 15, 1977
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) NRL-8115		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Navigation
Grid lock
Radar integration
Sensor integration
Radar

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A closed-form solution was obtained for the problem of finding an object's location given ranges to four stations of known location. The method is simple, involving only simple algebra such as addition and multiplication and the evaluation of a few trigometric functions. Only three stations are required for the solution if some other a priori information is available such as height above the earth or the azimuth.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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CONTENTS

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A SIMPLE CLOSED-FORM SOLUTION OF A POSITION-FIXING PROBLEM

INTRODUCTION

The problem of finding the location of an object is a part of the age-old navigation problem. Since electronics was introduced, accurate range measurements became possible, and, if the range difference is known from an object to several transmitter stations whose positions are accurately known, the location of the object can be determined by triangulation. Examples of systems which use range differences are Loran and Omega. In radar systems, ranges to an object can be found by either the object receiving echoes from known locations or by several known locations having radars receiving echoes from the object. The ranges are known in either case. These could be converted to range differences to correspond to the same problem as Loran and Omega. However, this is not necessary.

A simple means of obtaining the object position is described in this report. (An error analysis is planned for a future report.) The solution is in closed form and involves no more than simple algebra (adding, subtracting, multiplying, dividing, and taking square roots) and the evaluation of a few trigonemtric functions (sin, cos, arctan). The solution proceeds in several steps. First all stations of known location (which are noncoplaner) are specified in a common Cartesian coordinate system with the origin located at one of the station's positions. This coordinate system is then rotated such that a special form of the equations are obtained. The object's location is then found by simple algebra, and this solution is then passed through the inverse coordinate transformations to obtain the object's position.

STANDARD COORDINATE SYSTEMS AND TRANSFORMATIONS

Before the solution can be obtained, a means of describing the stations and object's position on the earth is reviewed. Figure 1 shows several earth-based coordinates. The definition of the geocentric coordinates (X, Y, Z), spherical representation (latitude θ, longitude λ, and radius vector ρ (earth's radius plus object height above earth)), and local coordinates (x, y, z) are well known and are not elaborated on further.

The transformations leading from one coordinate system to another are as follows. To convert from one local coordinate system to the geocentric coordinates, the transformation used is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = T \begin{bmatrix} x \\ y \\ z \end{bmatrix} + U,$$

Manuscript submitted March 4, 1977.

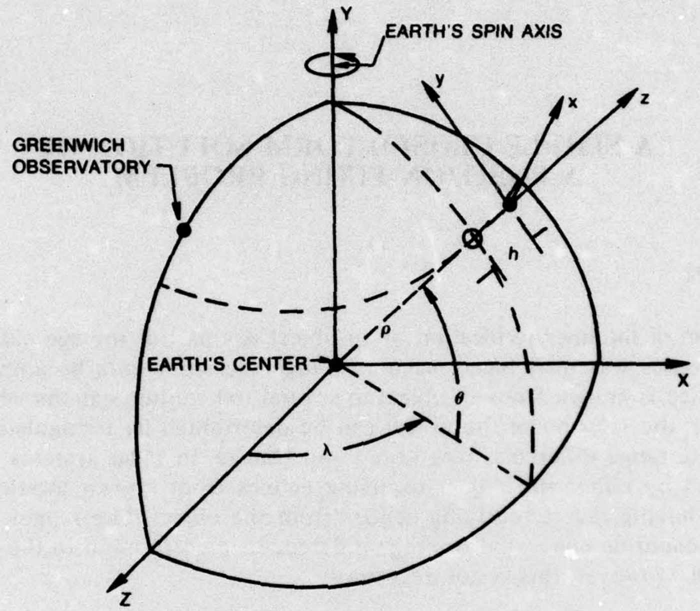

Fig. 1 — Coordinate systems defined on the earth.

where

$$T = \begin{bmatrix} \cos\lambda & -\sin\theta\sin\lambda & \cos\theta\sin\lambda \\ 0 & \cos\theta & \sin\theta \\ -\sin\lambda & -\sin\theta\cos\lambda & \cos\theta\cos\lambda \end{bmatrix}$$

and

$$U = \begin{bmatrix} \rho & \cos\theta\sin\lambda \\ \rho & \sin\theta \\ \rho & \cos\theta\cos\lambda \end{bmatrix}.$$

The inverse is found by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = T' \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - V,$$

where

$$V = \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix}.$$

The transpose of T denoted by T' is also the inverse of T.

FORUMLATION OF THE PROBLEM OF POSITION FIXING BY RANGING

It is assumed that the positions denoted by $(\theta_i, \lambda_i, \rho_i)$ for $i = 1$ through 4 for four stations and the ranges for $i = 1$ through 4 from these four stations to an object of unknown location $(\theta_p, \lambda_p, \rho_p)$ are known. The problem is to find a simple means of obtaining the objects' unknown location $(\theta_p, \lambda_p, \rho_p)$.

First the coordinates of the stations' position are converted to a C rtesian coordinate system. Specifically, the geocentric coordinates are used, and the coordinate transformation is

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = U_i = \begin{bmatrix} \rho_i & \cos\theta_i \sin\lambda_i \\ \rho_i & \sin\theta_i \\ \rho_i & \cos\theta_i \cos\lambda_i \end{bmatrix}$$

The four equations describing the ranges from each of the four stations to the unknown object are

$$(X_p - X_i)^2 + (Y_p - Y_i)^2 + (Z_p - Z_i) = r_i^2, \qquad i = 1, \ldots, 4. \tag{1}$$

The simultaneous solution of these four equations yields a unique position (X_p, Y_p, Z_p). However the equations in the form of (1) are difficult to solve. The solution discussed subsequently relies on a coordinate transformation to place the equations in an easily solvable form. The solution is then transformed back to obtain the desired answer.

SOLUTION

The first transformation in obtaining the solution is to transform the geocentric coordinates of the station to the local coordinates of one of the stations such that it is the origin of the new Cartesian coordinates. This is

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = T_1' \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} - V_1, \qquad i = 1, \ldots, 4,$$

where

$$T_1' = \begin{bmatrix} \cos\lambda_1 & 0 & -\sin\lambda_1 \\ -\sin\theta_1 \sin\lambda_1 & \cos\theta_1 & -\sin\theta_1 \cos\lambda_1 \\ \cos\theta_1 \sin\lambda_1 & \sin\theta_1 & \cos\theta_1 \cos\lambda_1 \end{bmatrix}$$

and

$$V_1 = \begin{bmatrix} 0 \\ 0 \\ \rho_1 \end{bmatrix}.$$

The next coordinate transformation is a rotation such that station 1 is the origin, station 2 lies on the new x axis (s axis), and station 3 lies on the new xy plane (st plane). The equation describing the rotation is

$$\begin{bmatrix} s_i \\ t_i \\ u_i \end{bmatrix} = Q \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \tag{2}$$

where

$$Q = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & \sin\beta \\ -\sin\alpha\cos\gamma - \cos\alpha\sin\beta\sin\gamma & \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & \cos\beta\sin\gamma \\ \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma & -\cos\alpha\sin\gamma - \sin\alpha\sin\beta\cos\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

in which

$$\alpha = \tan^{-1}(y_2/x_2),$$

$$\beta = \tan^{-1}\left[z_2/\sqrt{x_2^2 + y_2^2}\right],$$

and

$$\gamma = \tan^{-1} \frac{x_3 \cos\alpha \sin\beta + y_3 \sin\alpha \sin\beta - z_3 \cos\beta}{x_3 \sin\alpha - y_3 \cos\alpha}.$$

The details of the transformation (2) are discussed in Appendix A. The transformations used so far can be collected as:

$$\begin{bmatrix} s_i \\ t_i \\ u_i \end{bmatrix} = QT_1' \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} - V_1, \qquad i = 1, \ldots, 4, \tag{3}$$

where

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} \rho_i & \cos\theta_i \sin\lambda_i \\ \rho_i & \sin\theta_i \\ \rho_i & \cos\theta_i \cos\lambda_i \end{bmatrix}.$$

Equations (1) after transformation are in the form

$$(s_p)^2 + (t_p)^2 + (u_p)^2 = r_1^2, \tag{4}$$

$$(s_p - s_2)^2 + t_p^2 + (u_p)^2 = r_2^2, \tag{5}$$

$$(s_p - s_3)^2 + (t_p - t_3)^2 + u_p^2 = r_3^2, \tag{6}$$

and

$$(s_p - s_4)^2 + (t_p - t_4)^2 + (u_p - u_4)^2 = r_4^2. \tag{7}$$

These equations can easily be solved for the unknown position (s_p, t_p, u_p). From (4)

$$t_p^2 + u_p^2 = r_1^2 - s_p^2.$$

This is substituted into (5), yielding

$$s_p^2 - 2s_p s_2 + s_2^2 + r_1^2 - s_p^2 = r_2^2,$$

which can be solved for

$$s_p = \frac{r_1^2 - r_2^2 + s_2^2}{2s_2} = e. \tag{8}$$

Then (8) is substituted into (4) and (6), yielding

$$t_p^2 + u_p^2 = r_1^2 - e^2$$

and

$$(t_p - t_3)^2 + u_p^2 = r_3^2 - (e - s_3)^2.$$

Eliminating u_p yields

$$t_p = \frac{s_3^2 + r_1^2 - e^2 - r_3^2 + (e - s_3)^2}{2s_3} = f. \tag{9}$$

Finally (8) and (9) are substituted into (4), yielding

$$u_p = \pm\sqrt{r_1^2 - e^2 - f^2}. \tag{10}$$

Equation (7) is used to determine which sign on (10) should be used. The sign is used which most closely makes (7) to be true. The solution involves no more than simple algebra to construct and only requires addition, substraction, multiplication, division, and square-root operations for implementation.

The solution can be interpreted geometrically as follows. Equations (4) through (7) are equations of spheres. The intersection of two spheres yields a circle, and the intersection of a circle and a sphere yields two points. The two points intersecting a sphere yield a singular point. The last equation deciding on the sign of u_p may not be necessary if other information is available.

After the solution the inverse transformations are performed to move the coordinates back to the original frame of reference. The inverse of (3) is

$$\begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = T_1 Q' \begin{bmatrix} s_p \\ t_p \\ u_p \end{bmatrix} + U_1 .$$

The latitude, longitude, and radius vector for the object's position is then given by

$$\theta_p = \tan^{-1} (Y_p / \sqrt{X_p^2 + Y_p^2}),$$

$$\lambda_p = \tan^{-1} (X_p / Z_p),$$

and

$$\rho_p = \sqrt{X_p^2 + Y_p^2 + Z_p^2}.$$

Flow Chart

The computations required for finding an object's location by knowing the range to four stations of known location is best summarized with a flow chart (Fig. 2). The Cartesian representation of the geocentric coordinates U_i of the stations are found, and the coordinates are rotated by T_1 and translated by V_1 so that the origin of the new coordinates is at station 1. The rotation matrix Q is computed so that stations on the new coordinates have station 1 at the origin, station 2 on the new x axis, and station 3 on the new xy plane. The coordinates of the object are obtained with simple algebra. Finally the object's position is passed through the inverse transforms to obtain its position in terms of latitude, longitude, and radius vector.

SUMMARY

A closed-form solution of the problem of obtaining the location of an object, given the ranges to it from four known locations, was achieved. The method requires only simple algebra (addition, subtraction, multiplication and division) and the simple evaluation of trignometric functions such as sin, cos, and arctan. Only three stations are required for the solution if some other a priori information is available such as height above the earth or azimuth measurement. An error analysis is planned for a future report.

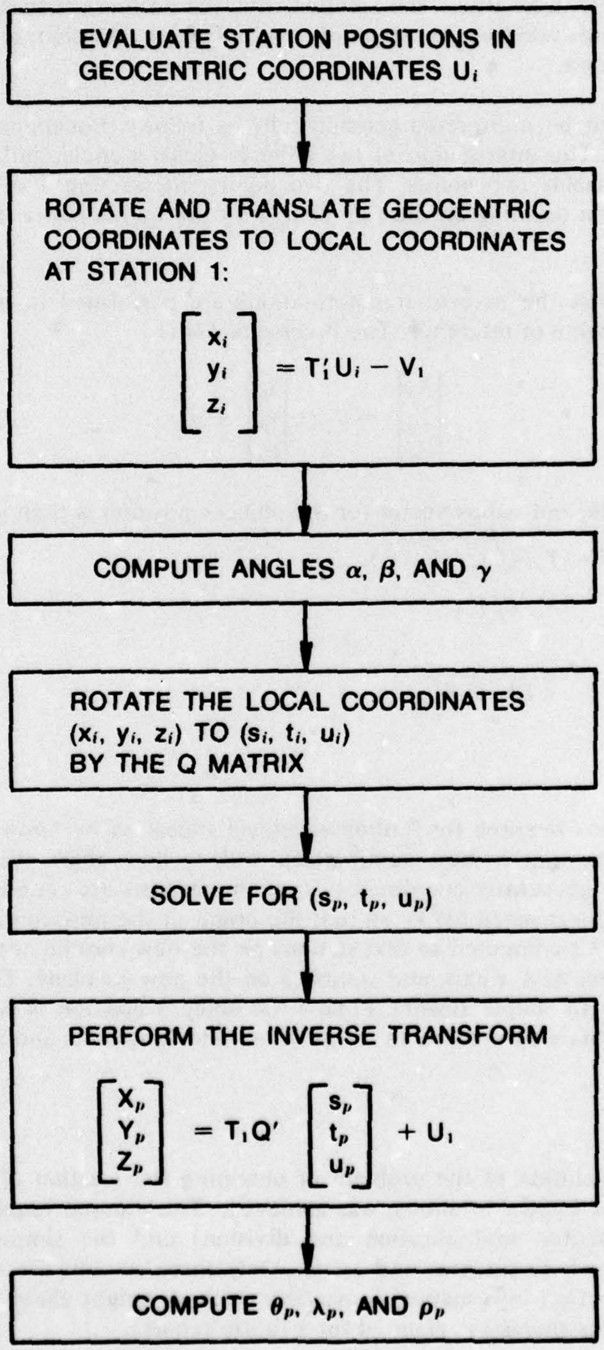

Fig. 2 — Flow chart of computations.

Appendix A
DETAILS OF THE COORDINATE ROTATION AS GIVEN BY EQ. (2)

Station 1 is defined at the origin initially. The problem is to first place station 2 on the new x axis after rotation. First a coordinate rotation by angle alpha about the z axis and then a rotation by angle beta about the new y axis is obtained. This two-step rotation is

$$\begin{bmatrix} x_i' \\ y_i' \\ z_i' \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & \sin\beta \\ -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha\sin\beta & -\sin\alpha\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}. \tag{A1}$$

Placing the coordinates (x_2, y_2, z_2) for station 2 into (A1), we desire y_2' and z_2' to be zero by definition. This yields

$$0 = -x_2 \sin\alpha + y_2 \cos\alpha \tag{A2}$$

and

$$0 = -x_2 \cos\alpha \sin\beta - y_2 \sin\alpha \sin\beta + z_2 \cos\beta. \tag{A3}$$

Solving (A2) for alpha, one obtains

$$\alpha = \tan^{-1}(y_2/x_2). \tag{A4}$$

Substituting (A4) into (A3), one then obtains

$$\beta = \tan^{-1} \frac{z_2}{\sqrt{x_2^2 + y_2^2}}. \tag{A5}$$

Finally the new coordinates of station 3 are required to not have a z component. A rotation about the x' axis by angle gamma is

$$\begin{bmatrix} s_i \\ t_i \\ u_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} x_i' \\ y_i' \\ z_i' \end{bmatrix}. \tag{A6}$$

Placing the coordinates of (x_3, y_3, z_3) of station 3 into (A6) and setting $u_i = 0$, one obtains

$$0 = -y_3' \sin\alpha + z_3' \cos\alpha$$

or

$$\gamma = \tan^{-1}(z_3'/y_3'),$$

which is from (A1)

$$\gamma = \tan^{-1} \frac{-x_3 \cos\alpha \sin\beta - y_3 \sin\alpha \sin\beta + z_3 \cos\beta}{-x_3 \sin\alpha + y_3 \cos\alpha}. \tag{A7}$$

The coordinate transformation Q is obtained by combining (A1) with (A6), with the angles being given by (A4), (A5), and (A7).